

**RELATIONSHIP BETWEEN SACK PERFORMANCE  
AND THE PROPERTIES OF SACK PAPER  
PART V. A Study of the Relationship between  
Uniaxial Tension Fatigue Life (Applied Energy)  
and the Progressive Height Sack Impact Test**

**Project 2033**

**Report Twenty-five**

**A Progress Report**

**to**

**MULTIWALL SHIPPING SACK PAPER MANUFACTURERS**

**October 31, 1962**

THE INSTITUTE OF PAPER CHEMISTRY

Appleton, Wisconsin

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LIFE (APPLIED ENERGY) AND THE PROGRESSIVE HEIGHT SACK IMPACT TEST

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MEMBERS OF GROUP PROJECT 2033

Albemarle Paper Manufacturing Co.

Continental Can Company, Inc.

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SUMMARY

1. The repeat impact performance of a multiwall sack appears to involve repetitive tension stresses in the sack paper. Work has been directed, therefore, to studying the uniaxial tension fatigue life of sack paper and its relationship to sack performance.
2. In an earlier phase of study, fatigue life was evaluated by controlling the strain (elongation) applied to the specimen in each cycle by means of an Instron testing machine. The result of this test is termed "strain fatigue life." Strain fatigue life compared favorably with conventional paper properties with respect to the relationship to sack performance, but the effect of humidity on sack behavior was not fully accounted for.
3. The sack impact test may be viewed as a stressing process involving application of particular amounts of energy to the paper of the sack by the commodity. Accordingly, control of the applied energy in each cycle of the Instron fatigue test may be a more meaningful way to evaluate uniaxial fatigue life. The result of this type of test is termed "energy fatigue life."
4. The materials of the second fabrication program (outer ply only) were evaluated for energy fatigue life at 50% R.H. to determine whether this property offers an improvement over strain fatigue life with respect to prediction of sack performance.

5. The energy fatigue lives of the regular paper in the two principal directions of the sheet were about equal, on the average, reflecting the fact that, although the cross-direction virgin energy is greater than the machine-direction virgin energy, more of it is nonrecoverable under tension cycling.
6. The machine-direction energy fatigue life was about twice the cross-direction life for the extensible papers, on the average, paralleling the ratio of virgin energies in the two directions.
7. On the average, machine-direction fatigue life of the extensible papers was twice that of the regular papers (virgin energies were in the ratio of about 4:1). Cross-direction fatigue lives of the two types of papers were more nearly equal, on the average, in keeping with their more nearly equal virgin energies.
8. Energy fatigue life and strain fatigue life in the machine direction were highly correlated, although the relationship appeared to differ between regular and extensible papers.
9. In the cross direction there was no significant correlation between energy fatigue life and strain fatigue life.
10. Energy fatigue life was slightly more variable within a sample than was strain fatigue life.
11. Machine-direction energy fatigue life was reasonably well correlated with sack performance (average differences of 11.7 to 14.8%) and gave comparable precision to strain fatigue life.

12. The relationship between cross-direction energy fatigue life and sack performance exhibited good precision (9.9 and 12.9% average difference) only when regular and extensible papers were considered separately. These relationships offered a modest improvement over strain fatigue life.
13. The most precise single relationship between sack performance and energy fatigue life was obtained with cross-direction life for the regular papers and machine-direction life for the extensible papers; the average difference was 12.7%.
14. Multiple linear regressions involving energy fatigue life in both directions of the paper offered precision of 9.0 to 13.0% and thus were comparable to strain fatigue life.
15. A product of power functions involving energy fatigue lives in both directions of the paper yielded precision virtually identical with the simpler multiple linear regressions, as was experienced earlier with strain fatigue life.
16. Neither energy fatigue life nor strain fatigue offered a clear-cut advantage over the other with respect to precision in predicting sack performance. The better relationship for combined regular and extensible papers was obtained with strain fatigue life (average difference of 11.8%); on the other hand, the better relationships for regular and extensible papers separately were achieved with energy fatigue life (average differences of 9.0 and 11.9%, respectively).
17. Although consideration of the mechanics of sack impact favor energy fatigue life over strain fatigue life, their relationships to sack performance at 50% R.H. were about equally precise for the samples of the second fabrication program.

18. The correlation between energy fatigue life and tensile energy absorption was high in the machine direction and only moderately good in the cross direction.
19. Energy fatigue life and Frag appeared to be related, although non-linearly, in the machine direction of the paper; there was no correlation apparent in the cross direction.
20. Neither strain fatigue life nor energy fatigue life offered an improvement over the conventional paper properties such as Thwing-Albert impact fatigue or tensile energy absorption, with respect to prediction of sack performance. All of the above-named paper tests ranked about equally, offering precision of the estimate of about  $\pm 9$  to  $\pm 12\%$  for the samples of the second fabrication program at 50% R.H.
21. In view of the time and skills required for determination of Instron uniaxial fatigue life, it cannot be recommended over Thwing-Albert impact fatigue or tensile energy absorption for purposes of evaluating the potential performance of sack paper.
22. The primary importance of the Instron uniaxial fatigue test appears to be as a research tool for studying the mechanism of the fatigue failure of sack paper. Laboratory fatigue testers such as the Thwing-Albert or Frag impact fatigue testers are not designed for such studies.



## INTRODUCTION

The repeat impact performance of a multiwall sack is by definition a fatigue phenomenon because the paper of the sack is repeatedly stressed from drop-to-drop until failure eventually occurs. Inasmuch as the sack is essentially flexible it would appear that the primary stresses induced in the sack paper are tension. For these reasons considerable attention has been directed to gaining an understanding of the tension fatigue behavior of kraft sack paper, from both a theoretical and experimental standpoint, and its relationship to sack performance (1-4).

The most recently reported work in this area of study was concerned with the relationship between progressive height sack impact performance and the uniaxial tension fatigue life of the parent paper evaluated by a progressively increasing applied strain program with an Instron testing machine (1). The work was performed on the materials from the second fabrication program at 50, 25, and 10% R.H. On the basis of empirical correlations with sack performance it was found that fatigue life compared favorably with conventional paper properties (Thwing-Albert impact fatigue, tensile energy absorption and Frag) with respect to predicting sack performance at 50% R.H. On the other hand, the uniaxial fatigue lives did not fully account for the effect of humidity on sack performance; an attempt at correcting for the possible effect of humidity on commodity behavior did not rectify the discrepancies. Thus, while Instron fatigue life compared favorably with conventional paper properties in predicting sack performance, the empirical relationships obtained were neither fully credible nor inclusive.

Fatigue life is not a single-valued property of sack paper; that is, there is no one value of fatigue life for a given sample of paper (at a given rate of testing and in a given environment) as is the case for virgin stretch, tensile

energy absorption, or tensile strength. There are many different ways of performing the fatigue test with an Instron testing machine--for example, controlling the elongation in each cycle versus the energy versus the stress, or varying any of these quantities in some prescribed manner from cycle to cycle versus maintaining a constant input from cycle to cycle. The various methods of performing the fatigue test on a sample of paper may be expected to lead to different results, in general.

Moreover, for any given type of controlled input the magnitude of the input may be selected from a wide range of values. Depending on the input magnitude, the fatigue life of the paper may vary from zero to an indefinitely large number. Complicating the matter is the fact that the relationship between the fatigue life and the magnitude of the input is highly nonlinear, thereby preventing simple proportioning to convert the results from one level of input to another.

In view of the foregoing, it is pertinent to inquire whether or not the results reported in Reference (1) could be materially improved if some method other than controlled strain had been used in performing the Instron fatigue tests. In particular, it has been suggested in various quarters (including those personnel conducting the studies) that controlling the energy input to the paper specimen may bear a closer analogy to sack impact than the applied strain process employed in Reference (1). This suggestion stems from the popular concept that on any given impact in the laboratory sack drop test the paper at a given location in the sack is subjected to a particular level of energy by the action of the contents, and that all sacks of similar design will experience this same input. And conversely it may be reasoned that sacks fabricated from different papers will undergo different strain and stress inputs, in general, because if the energy is fixed the strain and stress will depend on the shape of the specific stress-strain curve for each paper and, therefore, will vary in general from paper to paper.

Thus, there is a strong argument for viewing the sack impact test as an applied energy process and conducting the Instron fatigue tests accordingly. Carrying the physical concept to its logical conclusion involves applying constant energy inputs to the Instron specimen for correlation with the constant height sack impact test and applying progressively increasing energy inputs for correlation with the progressive height sack impact test. This still leaves unanswered the question of what magnitudes of energy input should be specified for closest simulation of sack impact. In view of the scant information available on the conditions existing during impact, the question is probably most easily resolved by selecting input magnitudes that give fatigue lives of the same order of magnitude as the sack itself, as was done in the applied strain tests of Reference (1).

The present study was carried out to determine whether or not evaluation of Instron fatigue life by controlling the energy input leads to a more precise relationship with sack performance than the applied strain process reported in Reference (1). For this purpose all runs of sacks from the recent fabrication program were evaluated at 50% R.H.

For ease of semantics the term "energy fatigue life" is frequently employed to designate Instron uniaxial fatigue life determined by controlling the energy applied during each cycle. Similarly "strain fatigue life" refers to the controlled strain tests.

### MATERIALS

The materials for this study were obtained from the second fabrication program which was carried out at Union Bag-Camp Paper Corporation, Savannah, Georgia, in December, 1960 (5). This fabrication involved 26 runs of 3-ply, pasted cement sacks manufactured from 50-lb. unbleached, kraft sack paper--twelve runs of regular paper and fourteen runs of extensible paper.

Samples of the sack paper were procured at the start and end of each run of sacks during fabrication and provided the supply of materials for the evaluation of Instron fatigue life.

## TEST PROCEDURE

### UNIAXIAL FATIGUE LIFE TESTS

Repeated tension tests were performed by means of an Instron testing machine equipped with line clamps (6). The specimen span was six inches, the width one inch, and the deformation rate (crosshead speed) was 0.2 inch/minute for both loading and unloading.

Six machine-direction specimens and six cross-machine direction specimens were tested for each run of sacks. The tensile specimens correspond to the outer ply of the fabricated sack. Three specimens from each group of six came from the start of the production run of sacks and three from the end of the run. Each set of three specimens was comprised of one specimen from the front of the parent roll, one from the center, and one from the back of the roll.

The repeated tension test was conducted by applying a prescribed energy during each cycle of loading. That is, the crossheads of the testing machine were moved apart until the applied energy indicated by an attached integrator reached a prescribed value and then the crossheads were reversed. A ten-second recovery period was observed between cycles.

The amount of energy applied in each cycle was progressively increased from cycle to cycle, as shown in Table I. For example, each specimen was subjected to an applied energy of 0.10 in.-lb./sq. in. on the first cycle, 0.12 in.-lb./sq. in. energy on the second cycle, etc. Cycling was continued until the specimen broke. The number of safe cycles (i.e., not including the cycle during which rupture occurred) is, by definition, the fatigue life of the specimen.

In any given cycle the maximum permissible deviation of the applied energy from the prescribed value was  $\pm 3\%$ ; if this deviation was exceeded the test was

repeated. In most instances the actual deviation was substantially less than 3%--  
generally less than  $\pm 1\%$ .

TABLE I

SCHEDULE OF APPLIED ENERGY IN REPEATED TENSION TESTS

Cycle Number	Applied Energy, in.-lb./sq. in.
1	0.10
2	0.12
3	0.14
4	0.16
5	0.18
6	0.20
...	...
<u>n</u>	$0.10 + (\underline{n}-1) (0.02)$

The schedule of applied energy listed in Table I was arrived at from the following considerations. Progressively increasing applied energy is believed to be analogous to the progressive height sack impact test, whereby each successive drop imparts an increasing magnitude of energy to the sack. The prescribed values of energy listed in Table I were selected on the basis of preliminary trials on the sample of regular paper having the lowest M.D. tensile energy absorption (T.E.A.) and on the extensible paper with the highest M.D. T.E.A. It was found that the energy input schedule gave a reasonable range of fatigue lives for these samples and thus could be expected to be suitable for both principal directions of all samples of this study. It may be of interest to note that the first application of energy (0.10 in. lb./sq. in.) represents about 38% of the M.D. T.E.A. of the regular sample and 6% of the M.D. T.E.A. of the extensible sample employed in the trials.

The test procedure described above differs from that of the "applied strain" process reported in Reference (1) in the following ways: (a) the energy

input to the specimen was controlled rather than the elongation; (b) the same energy input schedule was used for both directions of the sack paper (no prior information of the energy ratio occurring in sack impact was available as in the case of applied strain); (c) the specimens were tested individually rather than in groups of three (it could not be assumed that the total energy applied to three specimens simultaneously is uniformly divided between the three specimens).

Fatigue life tests were performed on all runs of sack paper at 50% R.H. (room temperature) after standard conditioning.

#### SACK IMPACT TESTS

Thirty sacks from each run were subjected to a progressive height face impact test starting at two feet and progressing by six-inch increments of height. The testing was performed as a part of the fabrication program (5).

## DISCUSSION OF RESULTS

### EXPERIMENTAL DATA

The Instron uniaxial fatigue lives (applied energy process) of the sack paper and the sack fatigue lives (i.e., number of safe drops) in the progressive height face impact test are listed in Table II. Each entry of Instron fatigue life is the average of six determinations; sack fatigue life is the average from thirty sacks. Fatigue lives of the individual specimens are listed in Table VII of the Appendix.

It may be noted in Table II that, on the average, the machine- and cross-machine direction fatigue lives of the regular papers were about equal; namely, 5.6 and 5.4, respectively. This result is in contrast to the relative virgin energy (tensile energy absorption) in the two directions; namely, 0.328 and 0.466 in. lb./sq. in., respectively (5), wherein the cross-direction T.E.A. is about 40% greater than machine-direction T.E.A. Much of the virgin energy in the cross direction, however, is nonrecoverable upon cycling and the apparent advantage of this direction of the paper for energy absorption is relinquished in repeated tension.

The average fatigue lives in the machine and cross direction of the extensible papers were 10.9 and 6.1, respectively. These lives are in the ratio of 1.8, which is approximately in the ratio of virgin T.E.A. in the two directions; that is, the average T.E.A. in the machine direction was 1.245 in.-lb./sq. in. and in the cross direction was 0.576 in.-lb./sq. in.--a ratio of 2.2. Both directions of extensible paper exhibit large amounts of nonrecoverable energy and cycling apparently deteriorates the virgin energy about equally in both directions.



TABLE II

FATIGUE LIFE OF SACK PAPER AND MULTIWALL SACKS

(50% R.H.)

*In + Cross - Code Symbol = MT*

Run	Instron Fatigue Life, no. of energy applications		Sack Fatigue Life, no. of drops	
	In-Machine	Cross-Machine		
	Regular Sack Paper			
AA	7.5	5.7	13.2 ✓	8.2
BB	5.5	5.2	10.7 ✓	7.6
CC	5.0	6.0	11.0 ✓	8.4
DD	7.7	5.3	13.0 ✓	6.6
EE	5.0	3.7-30	8.7	5.2
FF	3.7-	3.7-	7.4	5.5
GG	5.0	6.7+	11.7	7.1
HH	4.0 41	5.7	9.7	6.8
II	6.0	6.2	12.2	7.5
JJ	7.8+	5.8	13.6	9.2
KK	4.3	5.5	9.8	6.1
LL	6.0	5.7	11.7	6.4
Average	5.6	5.4		7.0

Extensible Sack Paper

MM	9.3	8.0 +	17.3	13.0
NN	10.7	7.2	17.9	14.5
OO	11.8	6.8	18.6	16.1
PP	9.5	5.7	15.2	12.7
QQ	11.7	6.0	17.7	15.1
RR	13.3+	5.8	19.1	17.4
SS	10.0	4.8-	14.8	8.8
TT	10.7 41	5.3	16.0	10.5
UU	11.8	7.3	19.1	10.7
VV	11.3	7.0	18.3	15.1
WW	9.2 -	5.7	14.9	13.0
XX	11.7	4.5	16.2	11.4
YY	10.0	5.5	13.5	12.2
ZZ	11.2	6.3	17.5	14.3
Average	10.9	6.1		13.2

Comparing the two types of papers within a given direction, the extensible papers' fatigue life (10.9) in the machine direction was about twice that of the regular papers (5.6), whereas, their T.E.A. were in the ratio of nearly 4:1. This

result again reflects the relatively larger amount of nonrecoverable energy in the extensible papers than in the regular papers in the machine direction.

The cross-direction fatigue lives of the two classes of papers were more nearly equal (5.4 versus 6.1) in keeping with their more nearly equal T.E.A. (0.466 versus 0.576 in.-lb./sq. in.).

The above comparisons of fatigue lives between types of sack paper and between directions parallel the trends reported by Mappus (7) using a constant applied energy process with an Instron testing machine.

Comparison of the individual fatigue lives in Table VII of the Appendix with the analogous data for the applied strain process of Reference (1) will reveal that the scatter in test results was slightly higher with the applied energy process. In about 50% of the samples of the present study the fatigue lives within a sample ranged over two or less cycles [as compared with 80% of the samples in Reference (1)]. In 80% of the samples of the present study the range was three cycles or less, and in 95% of the cases the range was four cycles or less.

Figures 1 and 2 show graphically the relationship between the Instron fatigue lives evaluated by the two methods--the applied energy process and the applied strain process. The data for the applied strain process are taken from Reference (1). It may be seen in Fig. 1 for the machine direction that the two types of fatigue lives were quite highly correlated. Considering both types of papers, the correlation coefficient was 0.941, as shown in Table III along with the equation of best fit relating one fatigue life to the other. A regression analysis for the extensible papers by themselves yielded virtually the same equation. The regular papers are not well clustered around the line of best fit for all samples and indicate that a separate line of steeper slope may better

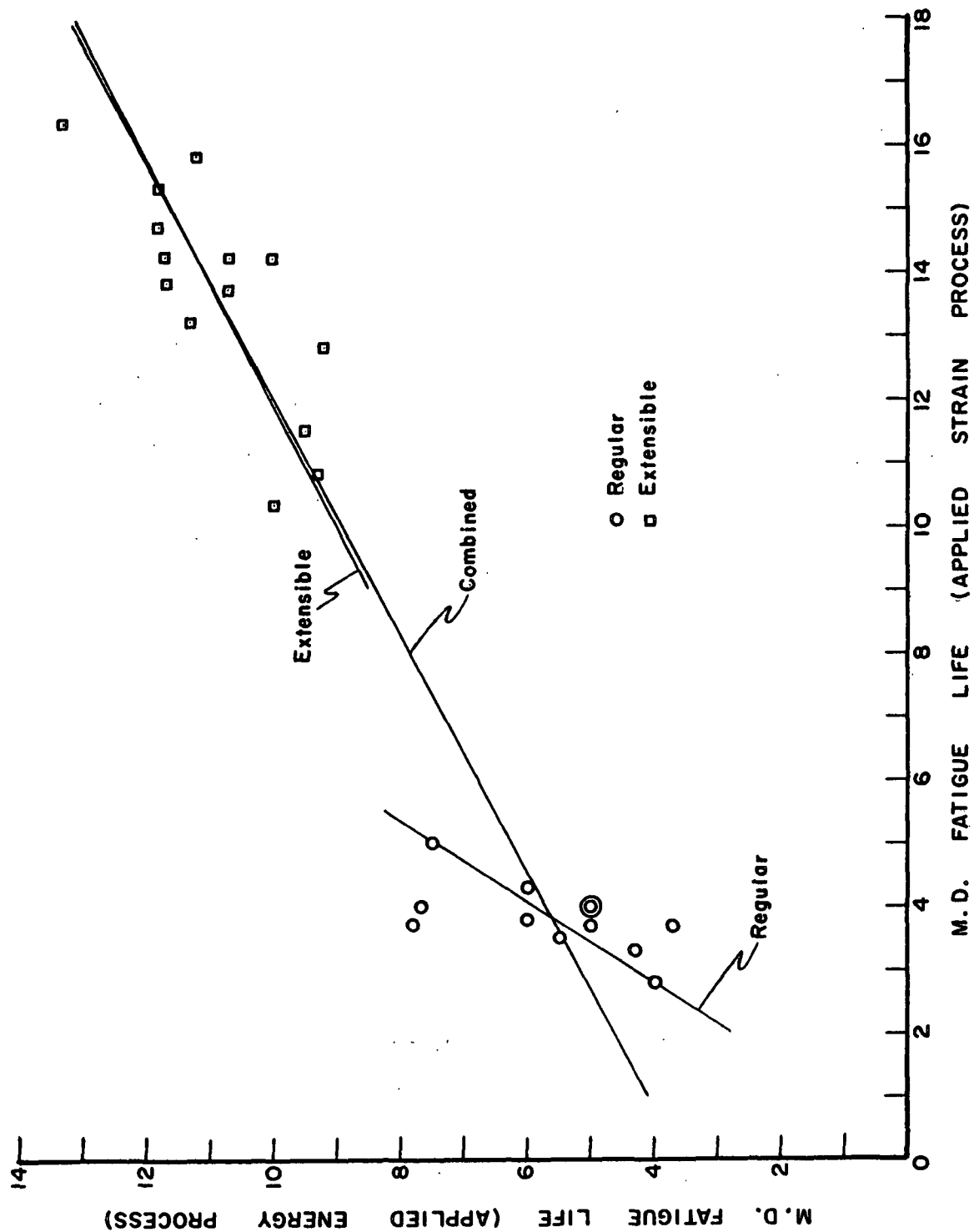


Figure 1. Relationship Between Machine-Direction Fatigue Lives Evaluated by Applied Energy Process and by Applied Strain Process at 50% R.H.

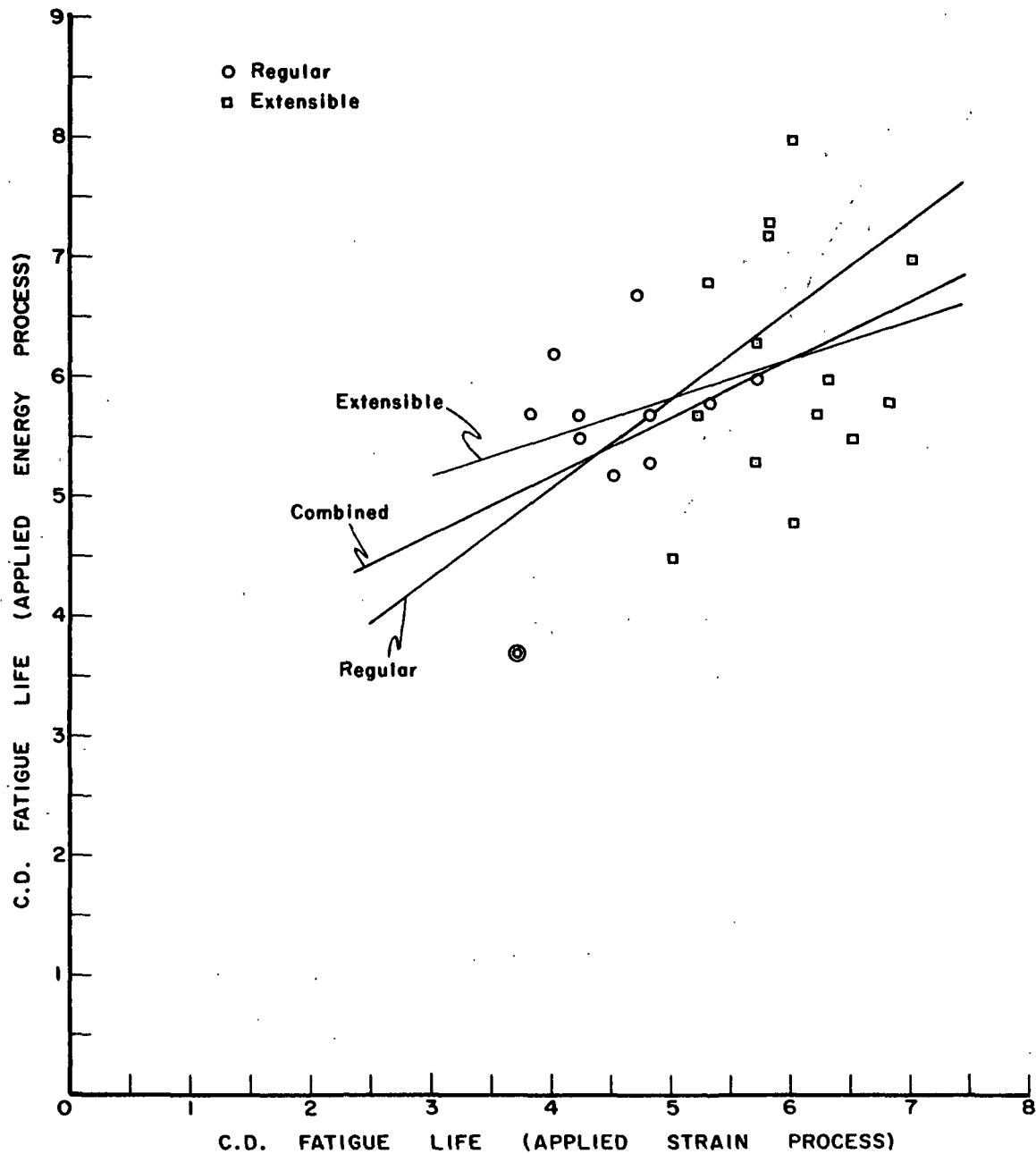


Figure 2. Relationship Between Cross-Direction Fatigue Lives  
Evaluated by an Applied Energy Process and by an Applied  
Strain Process at 50% R.H.

describe the relationship; however, the correlation coefficient for the regular papers alone is only 0.592, which is just barely significant at the 5% level.

TABLE III  
RELATIONSHIP BETWEEN FATIGUE LIVES EVALUATED BY THE APPLIED  
ENERGY AND APPLIED STRAIN METHODS  
(50% R.H.)

Direction	Type of Paper	No. of Observations	Correlation Coefficient	Equation <sup>a</sup>
In	Reg. & Ext.	26	0.941	$\underline{y} = 0.538\underline{x} + 3.55$
	Reg.	12	0.592	$\underline{y} = 1.563\underline{x} - 0.34$
	Ext.	14	0.778	$\underline{y} = 0.514\underline{x} + 3.87$
Cross	Reg. & Ext.	26	0.476	$\underline{y} = 0.496\underline{x} + 3.20$
	Reg.	12	0.532	$\underline{y} = 0.754\underline{x} + 2.08$
	Ext.	14	0.187	$\underline{y} = 0.327\underline{x} + 4.20$

<sup>a</sup>Symbols:  $\underline{y}$  = Energy fatigue life;  $\underline{x}$  = Strain fatigue life

Figure 2 and Table III reveal that there was virtually no correlation between the two types of fatigue lives in the cross-machine direction. The highest correlation coefficient was 0.532 (for the regular papers taken separately) and this is not significantly different from zero at the 5% level.

In summary, the "energy" fatigue life and the "strain" fatigue life were quite well correlated in the machine direction but very poorly correlated in the cross direction. If the two types of fatigue lives had been perfectly correlated (i.e., linearly related to each other), then the two types would correlate equally well with sack performance. Under the circumstances, however, it can be expected that the relationship between sack performance and energy fatigue life will differ somewhat from that reported earlier in Reference (1) for strain fatigue life.

## RELATIONSHIP BETWEEN UNIAXIAL FATIGUE LIVES AND SACK PERFORMANCE

The relationship between the observed Instron fatigue life (applied energy process) of the parent sack paper and the fatigue life of the sack (progressive height for drop) was studied by means of regression analysis. The forms selected are the same (though fewer in number) as were studied with respect to the applied strain process in Reference (1) and are (a) simple linear regression, (b) multiple linear regression and (c) product of power functions. The results of these analyses are listed in Table IV. In addition to the correlation coefficient the table lists the average (absolute) per cent difference between observed and predicted sack life, the latter based on the regression equation, and the distribution of differences in various percentage classes. The table also shows the salient features of the analogous regressions which were performed with strain fatigue life and reported in Reference (1).

### Simple Linear Regressions

As shown in Table IV, the correlation coefficient of the Regression No. 1 of sack performance on machine-direction fatigue life was 0.880 and the line of best fit has the equation,  $Z = 1.089X + 1.164$ , where  $Z$  = sack life and  $X$  = machine-direction Instron fatigue life. The average per cent difference between observed and predicted sack performance was 14.8%; 38% of the comparisons were within  $\pm 10\%$ , 69% were within  $\pm 20\%$ , and 88% were within  $\pm 30\%$ . The analogous regression involving strain fatigue life gave a correlation coefficient of 0.889 and an average difference of 13.3%. In this instance, no improvement was made in establishing the relationship between sack performance and paper properties by considering energy fatigue life.

TABLE IV  
REGRESSIONS OF SACK FATIGUE LIFE ON INSTRON FATIGUE LIFE  
(50% R.H.)

Independent Variables <sup>a</sup>	Type of Paper	No. of Observations	Energy Fatigue Life				Strain Fatigue Life					
			Regression No.	Correlation Coefficient	Average Diff., %	Per Cent of Differences Within + 10% + 20% + 30%	Regression No. <sup>b</sup>	Correlation Coefficient	Average Diff., %			
M.D. Fatigue Life, $\bar{X}$	Reg. & Ext.	26	1	0.880	14.8	38	69	88	$Z = 1.089\bar{X} + 1.164$	4	0.889	13.3
	Reg.	12	2	0.574	11.7	50	83	100	$Z = 0.483\bar{X} + 4.333$	7	0.271	13.1
	Ext.	14	3	0.497	14.6	36	71	86	$Z = 1.006\bar{X} + 2.267$	8	0.583	13.7
C.D. Fatigue Life, $\bar{Y}$	Reg. & Ext.	26	4	0.520	28.3	4	31	69	$Z = 1.887\bar{Y} - 0.604$	12	0.808	17.1
	Reg.	12	5	0.657	9.9	50	100	100	$Z = 0.871\bar{Y} + 2.319$	15	0.672	11.1
	Ext.	14	6	0.391	12.9	50	79	86	$Z = 0.921\bar{Y} + 7.551$	16	0.334	14.1
{ M.D. for Reg. } { C.D. for Ext. }	Reg. & Ext.	26	7	0.388	31.0	12	31	54	$Z = 1.165\bar{U} + 3.487$ $\bar{U} = \bar{X}$ for Regular $\bar{Y}$ for Extensible	17	0.850	38.1
	Reg. & Ext.	26	8	0.897	12.7	54	85	92	$Z = 1.110\bar{U} + 1.081$ $\bar{U} = \bar{Y}$ for Regular $\bar{X}$ for Extensible	18	0.743	47.1
M.D. and C.D. Fatigue Life, $\bar{X}$ and $\bar{Y}$	Reg. & Ext.	26	9	0.904	13.0	46	81	92	$Z = 0.986\bar{X} + 0.803\bar{Y} - 2.634$	22	0.916	11.8
	Reg.	12	10	0.767	9.0	67	100	100	$Z = 0.349\bar{X} + 0.707\bar{Y} + 1.244$	25	0.704	11.0
	Ext.	14	11	0.634	11.9	57	86	93	$Z = 1.010\bar{X} + 0.927\bar{Y} - 3.469$	26	0.650	12.4
M.D. and C.D. Fatigue Life, $\bar{X}$ and $\bar{Y}$	Reg. & Ext.	26	12	0.908	12.8	42	85	92	$Z = 0.936\bar{X} - 0.722\bar{Y} + 0.488$	36	0.922	11.9
	Reg.	12	13	0.796	9.3	50	100	100	$Z = 1.883\bar{X} - 0.245\bar{Y} + 0.547$	--	--	--
	Ext.	14	14	0.626	11.5	64	86	93	$Z = 0.891\bar{X} - 0.738\bar{Y} + 0.513$	--	--	--
Product of Power Functions												

<sup>a</sup> Symbols:  $Z$  = Sack fatigue life (no. of safe drops) in progressive height face drop;  $\bar{X}$  = machine-direction fatigue life;  $\bar{Y}$  = cross-direction fatigue life.  
<sup>b</sup> Numbering refers to Reference (1).

A graph of the relationship between sack performance and machine-direction energy fatigue life is presented in Fig. 3. The line of best fit for Regression No. 1 is also shown. It may be seen that five plotted points militate against a good linear relationship between the variables. These are Samples SS, TT, UU, and XX in the extensible papers and Sample DD in the regular papers. In all five instances the sack performance is appreciably lower than can be accounted for by machine-direction energy fatigue life.

Regressions No. 2 and 3 were performed on regular and extensible papers separately. In the case of the regular papers the line of best fit was substantially different from the combined regression and, although the correlation coefficient is just barely significant at the 5% level, the average difference was reduced to 11.7%. The regression line for the extensible papers separately is virtually the same as the combined regression and no appreciable change in per cent difference (namely, 14.6%) was encountered. The observation that the correlation coefficient decreased to a nonsignificant 0.497 with essentially no change in regression line nor per cent difference provides a marked example of the dependence of the coefficient on the range encompassed by the independent variable.

Taken in their entirety, Regressions 1, 2, and 3 involving machine-direction energy fatigue life appear to offer no improvement over the analogous regressions involving strain fatigue life.

Turning to consideration of the cross-direction fatigue life, Regressions 4, 5, and 6 may be inspected. The regression on the combined regular and extensible papers (No. 4) was relatively poor with a correlation coefficient of 0.520 and an average difference of 28.3%--both measures of precision being markedly inferior to those obtained with strain fatigue life. The reason becomes clear upon examination of Fig. 4 which is a graph of sack performance versus cross-direction energy



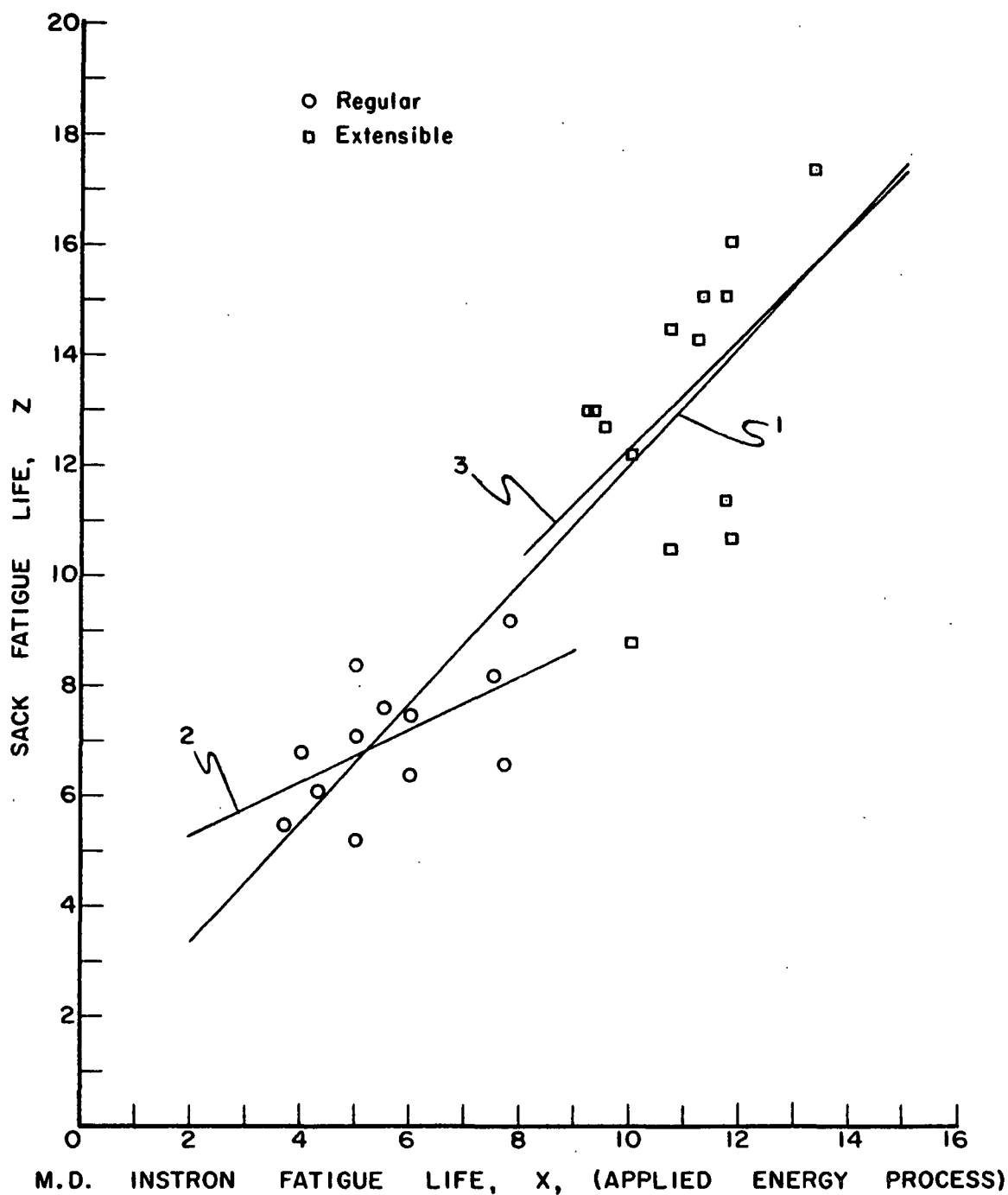


Figure 3. Relationship Between Sack Performance and Machine-Direction Instron Fatigue Life Evaluated by an Applied Energy Process at 50% R.H. (Regressions 1-3)

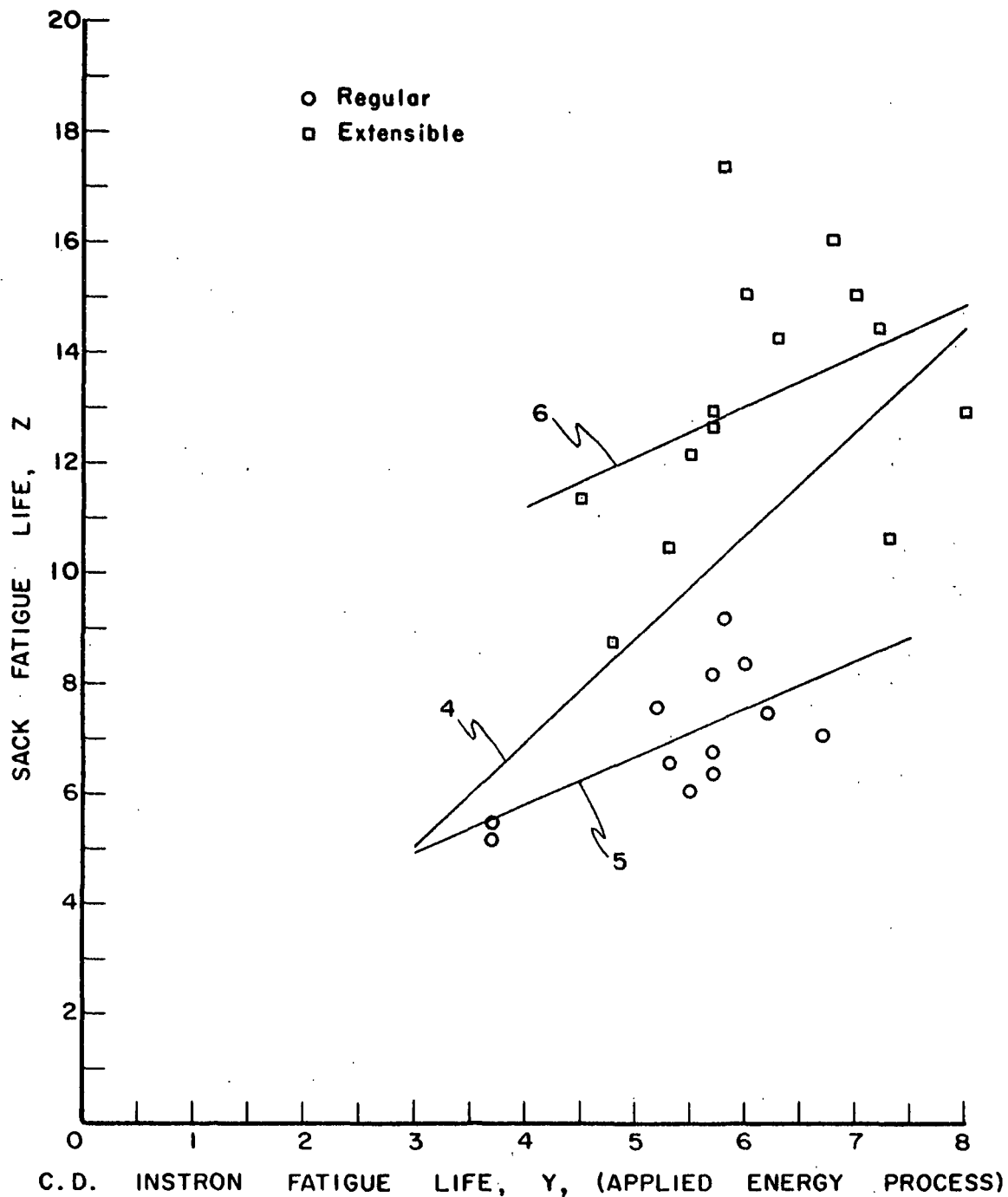


Figure 4. Relationship Between Sack Performance and Cross-Direction Instron Fatigue Life Evaluated by an Applied Energy Process at 50% R.H. (Regressions 4-6)

fatigue life. The plotted points for the extensible samples are arrayed essentially vertically above those for the regular paper. In other words, for a given level of paper fatigue life (cross direction) the two types of papers exhibit different levels of sack performance. Obviously, a single equation, such as Regression No. 4, cannot describe both classes of papers. Separate equations (with nearly equal slopes but differing intercepts) for regular and extensible papers markedly improved the predictive ability for each type of paper (9.9 and 12.9%), although the scattering of plotted points in either class leaves doubt as to the precision of the empirically determined constants of the equations. Much the same trend occurred with strain fatigue life, as shown in Fig. 4 of Reference (1).

In summary, a single equation was not effective in describing the relationship between sack performance and cross-direction energy fatigue life, although for either class of papers a slight improvement over strain fatigue life was achieved with separate equations for regular and extensible papers. Neither type of Instron fatigue life in the cross direction exhibited a very convincing relationship with sack performance, in the sense that by itself the cross-direction fatigue life did not account for major shifts in level of sack performance between the two types of paper.

Regression No. 7 was performed with machine-direction energy fatigue life for the regular papers and cross-direction fatigue life for the extensible papers. In Reference (1), this grouping of the data was termed the "weaker direction" analysis because it involved directions for which the paper properties were the lowest. The characterization is less appropriate for energy fatigue life because with regular papers the machine- and cross-direction fatigue lives were about equal, in general. Accordingly, Regression 7 yields results of about comparable precision to Regression 4, namely, per cent differences near 30%, on the average, and a low

correlation coefficient. The analogous analysis involving strain fatigue life was also unsatisfactory with an average difference of 38.1%.

Regression 8 was performed with cross-direction energy fatigue life for regular papers and machine-direction fatigue life for extensible papers. For the reason mentioned in the preceding paragraph, this regression is nearly the same as Regression 1 involving machine-direction fatigue life for both types of papers. Both the average difference, 12.7%, and the distribution of differences for Regression 8 were the most favorable of any of the simple linear regressions covering both types of paper. A graph of this relationship is given in Fig. 5. Comparison with Fig. 3 involving machine-direction fatigue life for both papers, indicates that Regression 8 is an improvement over Regression 1 because of a better relationship for the regular papers.

The regression with strain fatigue life analogous to Regression 8 was relatively poor with an average difference of 47.1%.

#### Multiple Regressions

In Regressions 9-11 equations of the form  $Z = AX + bY + c$  were studied, where  $Z$  = sack fatigue life,  $X$  = machine-direction Instron fatigue life, and  $Y$  = cross-direction Instron fatigue life. In Regression 9 the regular and extensible samples were combined. The average difference was 13.0% which is slightly less favorable than the similar regression employing strain fatigue life, namely, 11.8%. Regression 9 represents an improvement over the simple regressions involving only one direction of the paper (Regressions 1 and 4), as might be anticipated. Curiously, however, the precision of Regression 9 is not quite as favorable as Regression 8 which employed cross-direction fatigue life for regular papers and machine-direction life for the extensibles. The latter regression yielded an average difference of 12.7% and was superior to either Regression 1 or 4, as noted earlier.

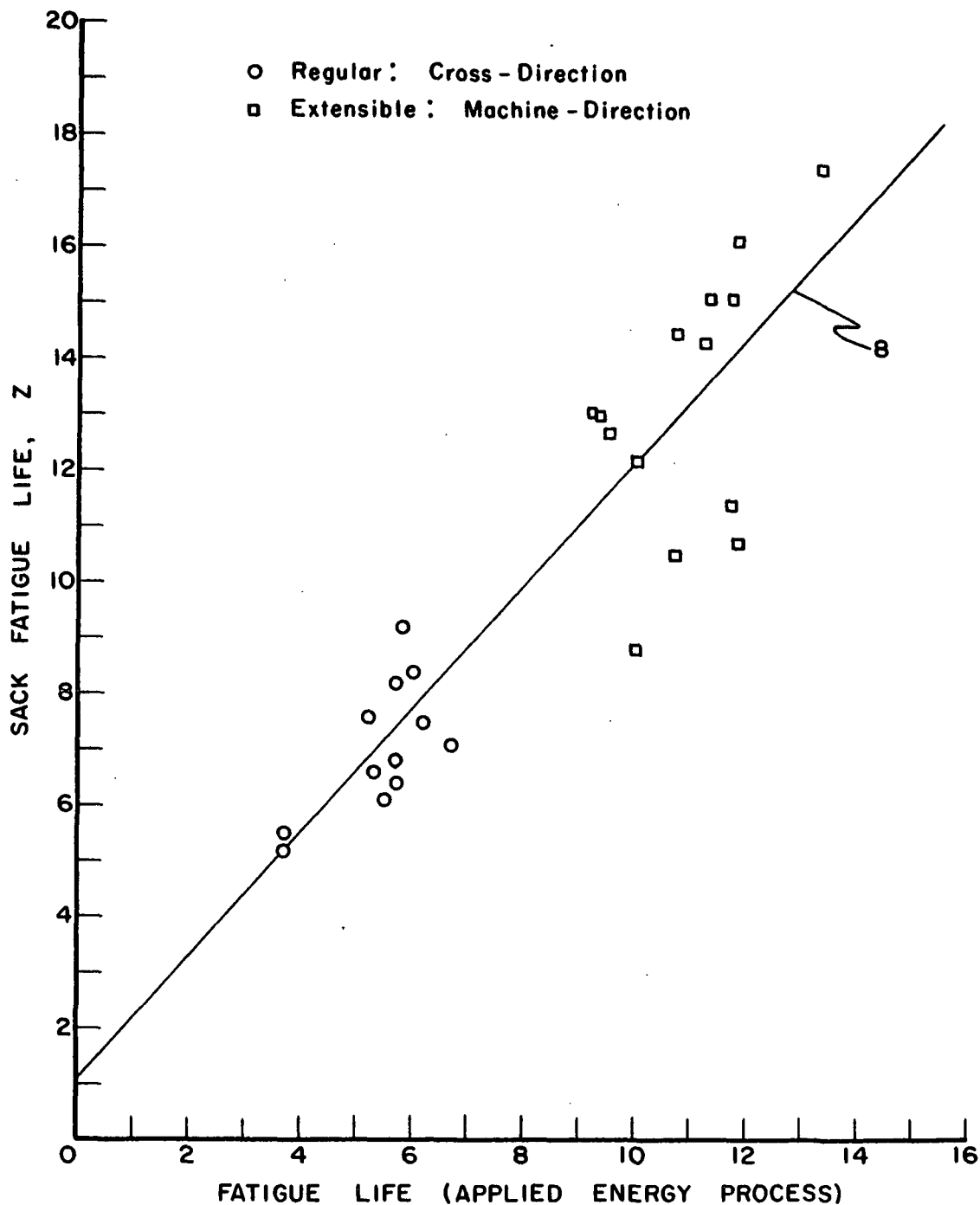


Figure 5. Relationship Between Sack Performance and Instron Fatigue Life at 50% R.H.--Cross-Direction for Regular Paper and Machine Direction for Extensible Paper (Regression 8)

Regressions 10 and 11 reveal that somewhat better precision was obtained by separating the regular and extensible papers, with average differences of 9.0 and 11.9%, respectively, which are modest improvements over strain fatigue life under the same division of samples. Inspection of the coefficients of the equations involving energy fatigue life indicate that the relationship for extensible papers alone is essentially the same as for the combined papers, but the relationship for regular papers alone is quite different. This observation seems to be a characteristic evidenced throughout this study.

Lastly, it may be seen in Table IV that a power function relationship between sack performance and Instron fatigue lives ( $Z = ax^b y^c$ ) yielded correlation coefficients and precision of the estimate which were virtually identical with the multiple linear regressions (compare Regressions 12-14 with Regressions 9-11). Thus, there is no advantage to be gained with these data in going to the seemingly more general equation involving power functions. The same conclusion was reached in Reference (1) with respect to strain fatigue life.

In summary, it may be seen in Table IV that neither type of fatigue life--energy or strain--offered a clear advantage over the other with respect to prediction of sack performance. The better relationship for combined regular and extensible sacks was obtained with strain fatigue life with an average difference of 11.8% between observed and predicted sack performance (as contrasted with 12.7% with energy fatigue life). On the other hand, the better prediction for regular papers alone was achieved with energy fatigue life, exhibiting an average difference of 9.0% (as compared with 11.0% for strain fatigue life). Similarly, for extensible papers alone, the relationship based on energy fatigue life incurred an average difference of 11.9% (as contrasted with 12.4% for strain fatigue life).

From a consideration of the mechanics of the sack impact test, energy fatigue life would seem to be more appropriate than strain fatigue life. With these samples, however, the effect of the underlying differences in the two methods of fatigue testing apparently is obscured by other factors.

Earlier work (1) showed that strain fatigue life offered an advantage over conventional paper properties with respect to prediction of sack performance. With the 26 samples of regular and extensible sacks at 50% R.H., the average per cent difference with strain fatigue life was 11.8%, while Thwing-Albert impact fatigue, tensile energy absorption and Frag gave precisions of 16.8, 19.1 and 20.9%, respectively. In these studies involving conventional paper properties, sack performance was expressed in terms of safe inches, whereas safe drops (fatigue life) was employed for the fatigue investigation. Subsequent work (8) revealed that the precision obtained with the conventional paper properties was materially improved by expressing sack performance in terms of safe drops. The precision achieved with Thwing-Albert impact fatigue was improved from 16.8 to 9.4% and the precision with tensile energy absorption improved from 19.1 to 10.2%. Both of these improved precisions are slightly better than was obtained with strain fatigue life. The present study indicates that the ranking of these several types of paper tests is not changed by replacing strain fatigue life with energy fatigue life.

It should be kept in mind that the precision achieved with strain or energy fatigue life probably would be improved somewhat by more extensive replication--only six specimens were tested per sample as contrasted with 36 or 72 specimens with the conventional paper tests. Nonetheless, it seems improbable that the precision of the estimate of sack performance could be markedly improved over that of the Thwing-Albert and tensile energy absorption. That is, it appears likely that the uniaxial fatigue properties would continue to offer precision

comparable to those of the conventional paper properties, rather than show a major advantage.

In view of the time and skills required to perform the Instron fatigue tests and the precision offered by uniaxial fatigue life, this paper property cannot be recommended over the conventional tests for purposes of ranking sack paper with respect to its potential performance in the sack. It appears that the value of the Instron fatigue test is primarily as a research tool rather than a paper evaluation test for control of quality. The high precision offered by the Thwing-Albert impact fatigue test (with respect to prediction of sack performance) attests to the importance of fatigue in sack behavior. (The nearly as high precision offered by tensile energy absorption can be attributed to its high degree of correlation with fatigue life, as shown in Reference (1) and later in the present report.) But the Thwing-Albert test leads to little or no understanding of the mechanism of fatigue failure in the paper--providing merely an index (fatigue life or safe drops) of its performance under a particular repeated stressing process. As shown in References (3) and (4) the Instron uniaxial fatigue life renders considerably more accessible an understanding of the mechanism of fatigue failure. For this reason it is believed that the concepts of uniaxial fatigue performance will find continued use in research studies on sack performance.

#### COMPARISON OF INSTRON FATIGUE LIFE AND CONVENTIONAL PAPER PROPERTIES

It may be recalled from Reference (1) that strain fatigue life was highly correlated with virgin stretch--a result that had been anticipated from the mathematical analysis of the mechanism of repeated tension (3, 4). Although the number of cycles that the paper can withstand also depends on the elastic and plastic slopes, reload slope, and proportional limit of the tension load-elongation curve, the dominant factor appears to be the virgin stretch.



Although the mathematical analysis of the applied energy process for repeated tension has not been formalized, it seems likely that the virgin energy (T.E.A.) may play a dominant role in determining the fatigue life under this type of uniaxial stressing. For this reason it seemed appropriate to investigate the degree of correlation between energy fatigue life and T.E.A. as reported in Reference (5).

The results of the correlation analysis are given in Table V and Fig. 6 and 7. In the machine direction, fatigue life and T.E.A. were highly correlated over both regular and extensible papers--a correlation coefficient of 0.961. Inspection of Fig. 6, however, indicates that the relationship probably differs between regular and extensible papers. Over each type of paper the correlation coefficient remains high (0.903 and 0.934). That a different relationship may be required for each type of paper is undoubtedly a result of the differing proportion of nonrecoverable energy in the two classes of paper, as mentioned earlier in this report. Or stated differently, the remaining parameters of the load-elongation curve that determine machine-direction fatigue life probably differ between regular and extensible papers. At any rate, it is apparent that with the equations given in Table V, the machine-direction fatigue lives of these samples could be predicted generally to within one cycle.

The correlation between cross-direction fatigue life and T.E.A. is not as high as with the machine direction. The correlation coefficients range from 0.75 to 0.80. The equations expressing the relationship are nearly the same for either or both types of paper, which suggests that the tension parameters governing cross-direction repeated tension may not vary widely between these two classes of papers. This appears reasonable because the distinctive characteristics of extensible papers are primarily associated with the machine direction. Comparison of Fig. 6 and 7

indicate that cross-direction fatigue life could not be predicted quite as accurately from T.E.A. as in the machine direction.

TABLE V  
RELATIONSHIP BETWEEN "ENERGY" FATIGUE LIFE AND TENSILE ENERGY ABSORPTION  
(50% R.H.)

Direction	Type of Paper	No. of Observations	Correlation Coefficient	Equation
In	Reg. & Ext.	26	0.961	$y = 5.81x + 3.67$
	Reg.	12	0.903	$y = 24.11x - 2.29$
	Ext.	14	0.934	$y = 5.64x + 3.85$
Cross	Reg. & Ext.	26	0.799	$y = 8.04x + 1.59$
	Reg.	12	0.758	$y = 8.00x + 1.70$
	Ext.	14	0.789	$y = 9.44x + 0.70$

Figures 8 and 9 show the relationship between energy fatigue life and Frag, where the latter is expressed in terms of number of safe drops rather than burst energy. The Frag test results are listed in Table VI and were obtained as a part of the fabrication program. Figure 8 suggests a curvilinear relationship for the machine direction as indicated by the visually fitted curve. The sense of the convexity may be attributable to the nature of the energy inputs in the two tests. In the Instron test the applied energy was increased from cycle to cycle whereas in the Frag the applied energy is constant. As the number of Frag drops increases the Instron performance falls off (from the linear) because the latter represents a more severe input to the paper.

There is little or no correlation apparent between the Instron fatigue life and Frag in the cross direction.

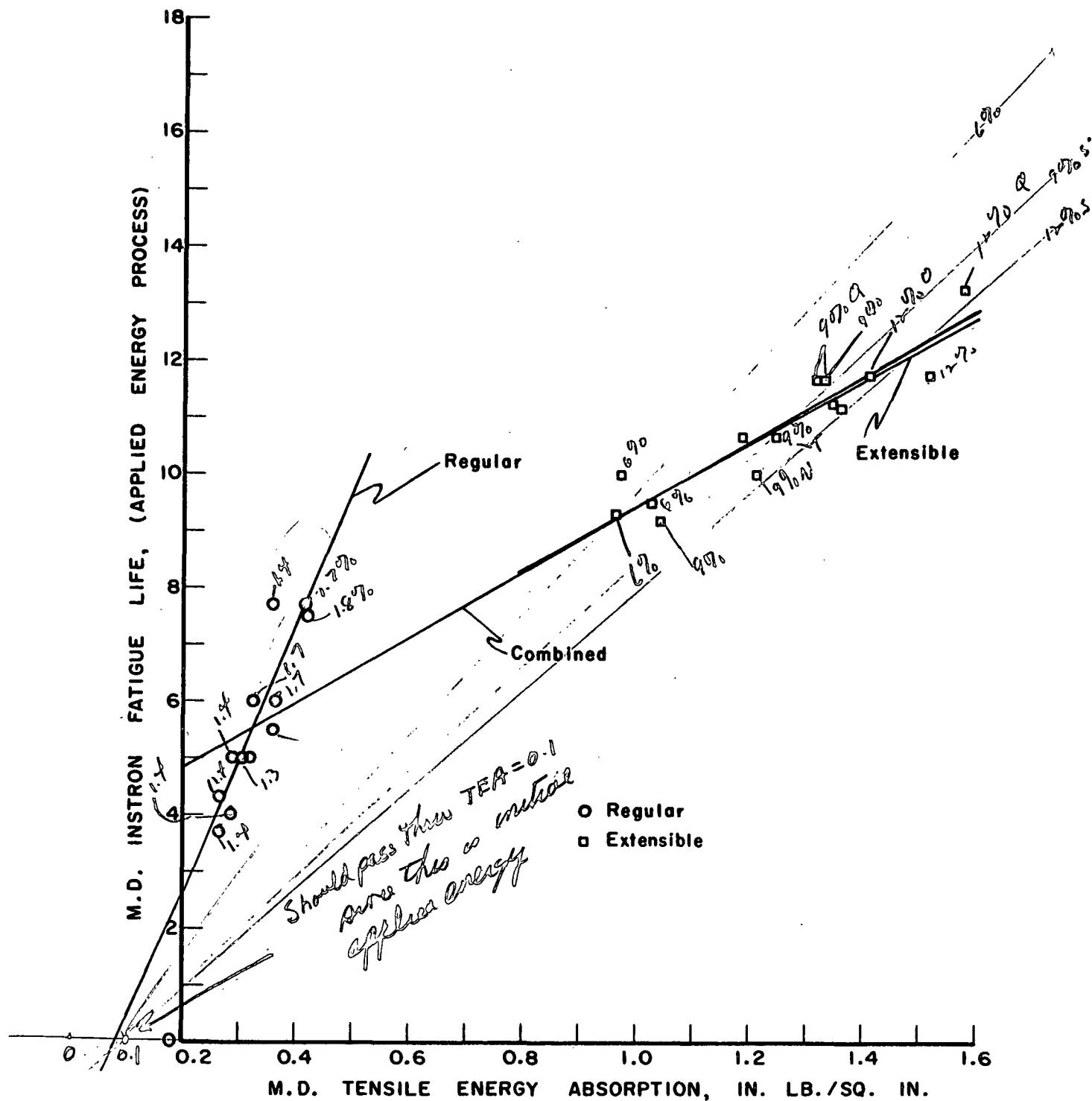


Figure 6. Relationship Between Machine-Direction Energy Fatigue Life and Tensile Energy Absorption at 50% R.H.

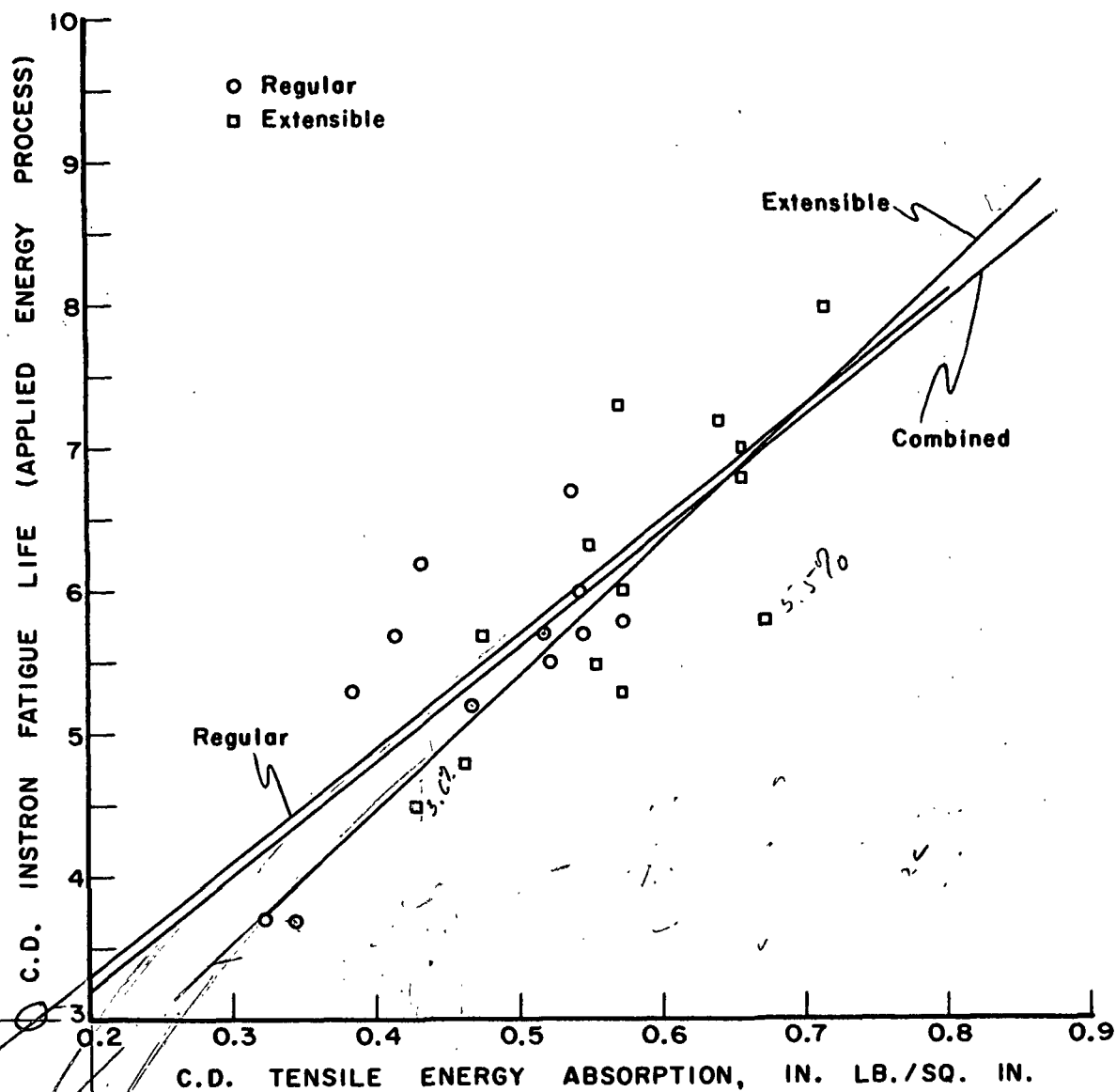


Figure 7. Relationship Between Cross-Direction Energy Fatigue Life and Tensile Energy Absorption at 50% R.H.

*should pass thru this point  
for initial level of  
applied energy*

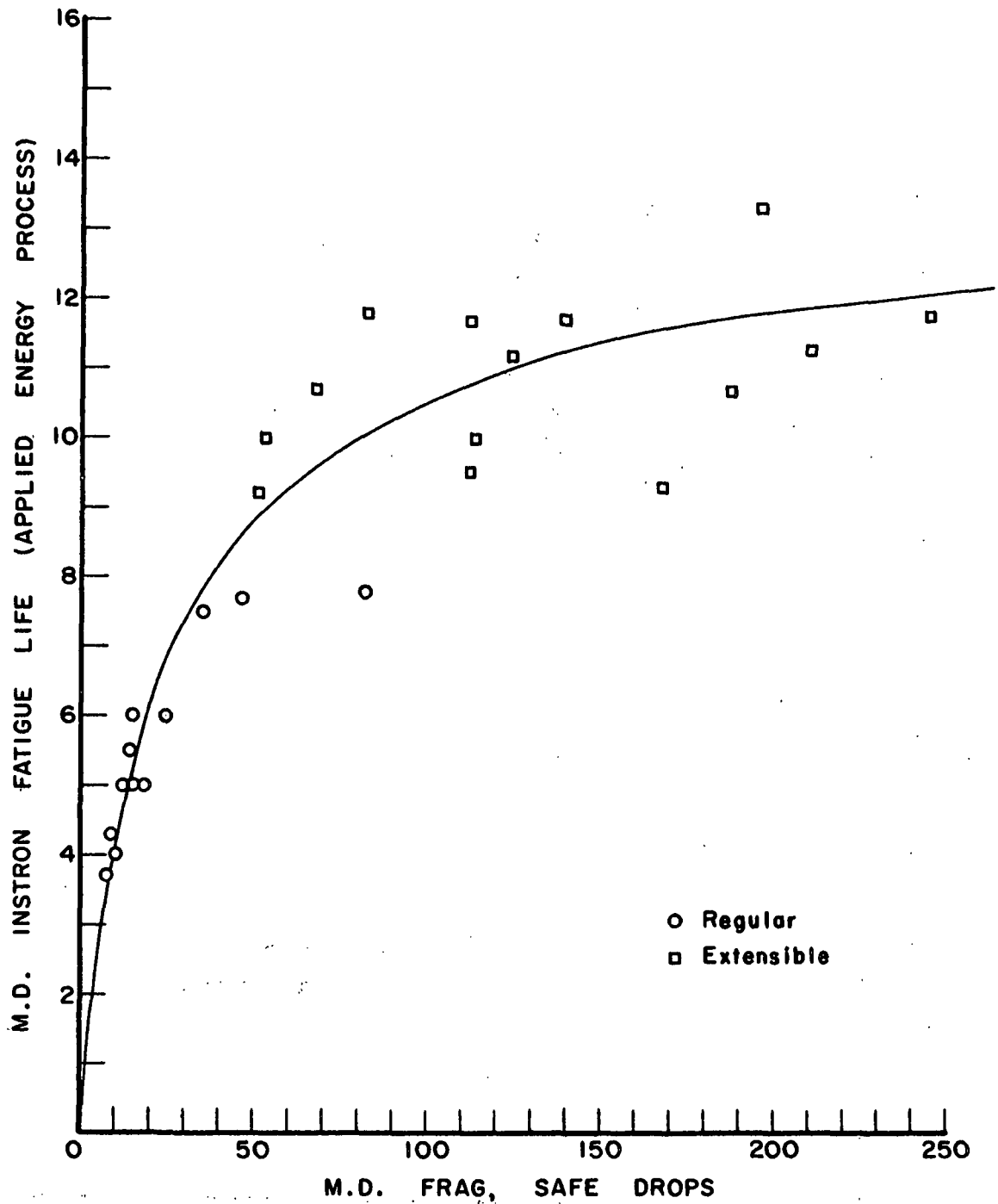


Figure 8. Relationship Between Machine-Direction Energy Fatigue Life and Frag at 50% R.H.

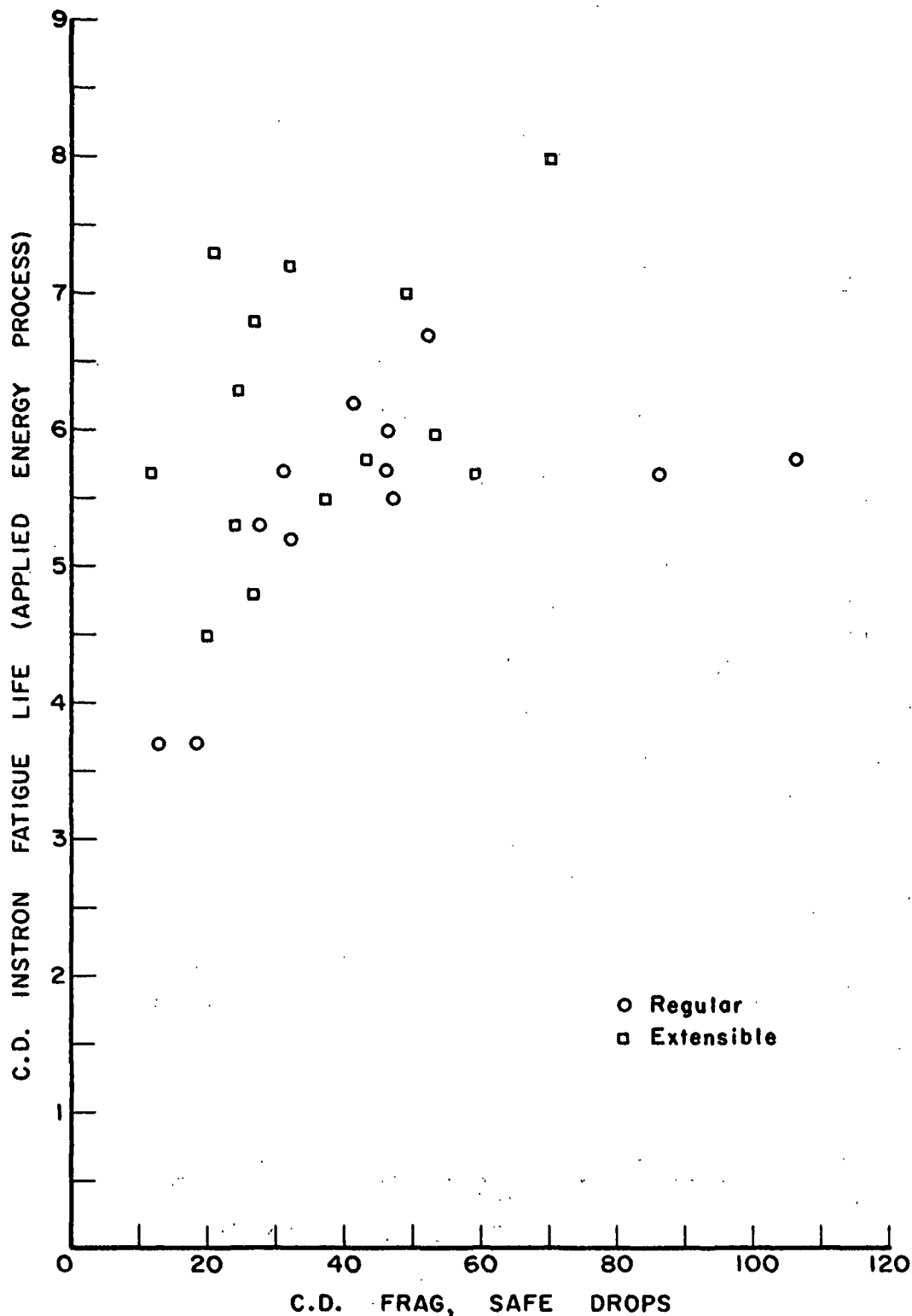
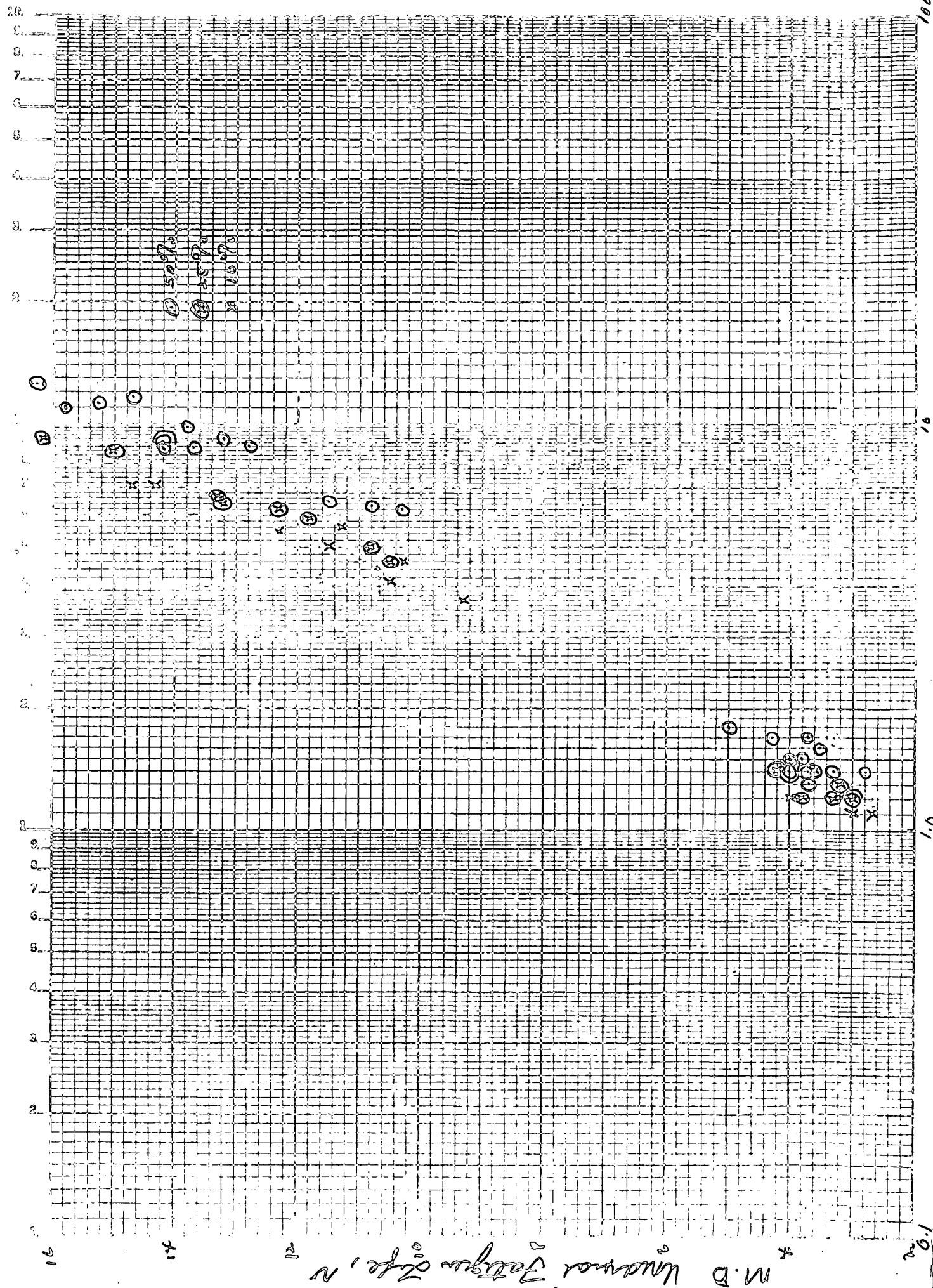


Figure 9. Relationship Between Cross-Direction Energy Fatigue Life and Frag at 50% R.H.

Stretch, % [Machine Direction]



not 7. (p. 1.0)



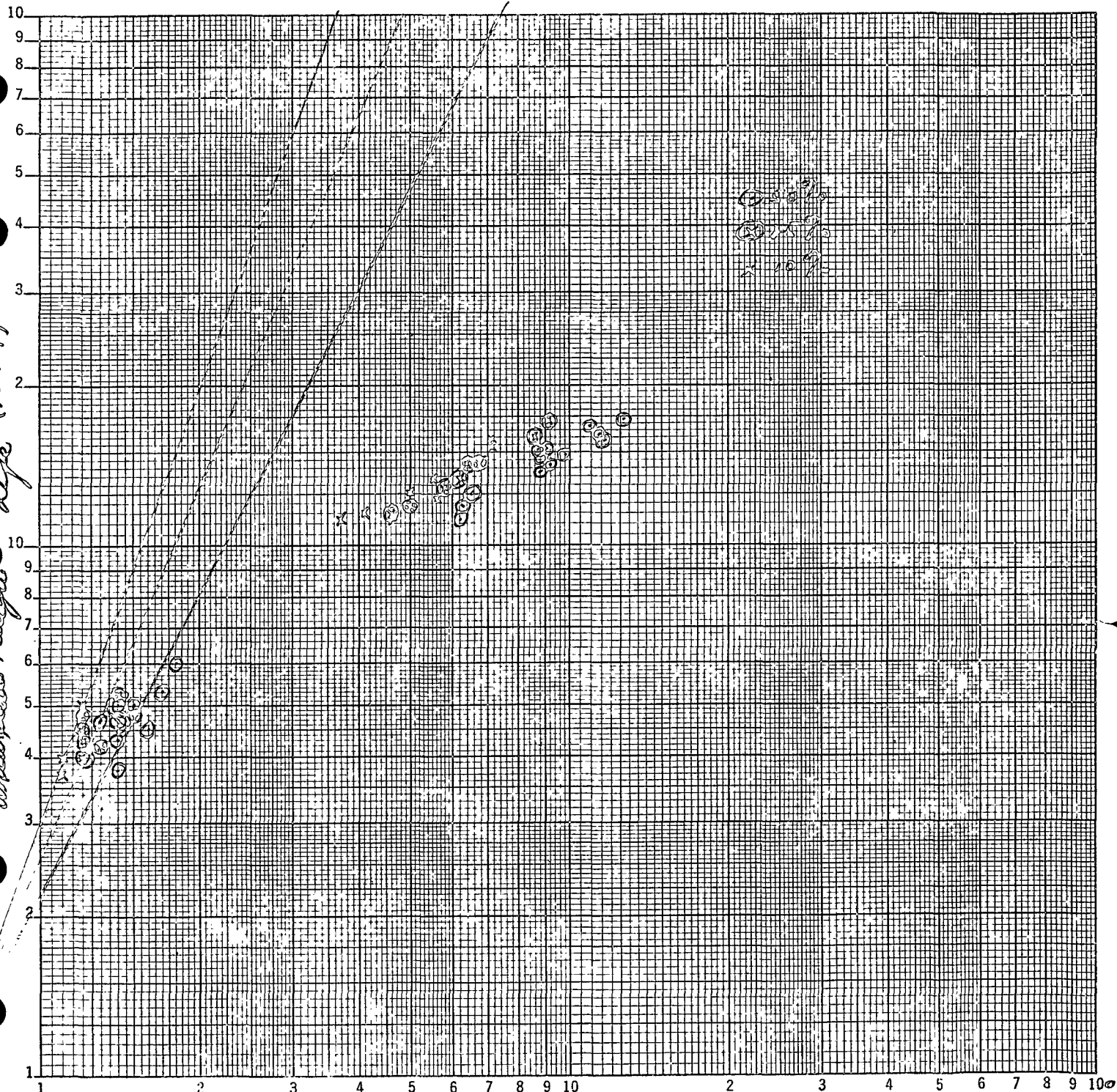
Logarithmic, 2 X 2 Cycle

Wye (N+1)

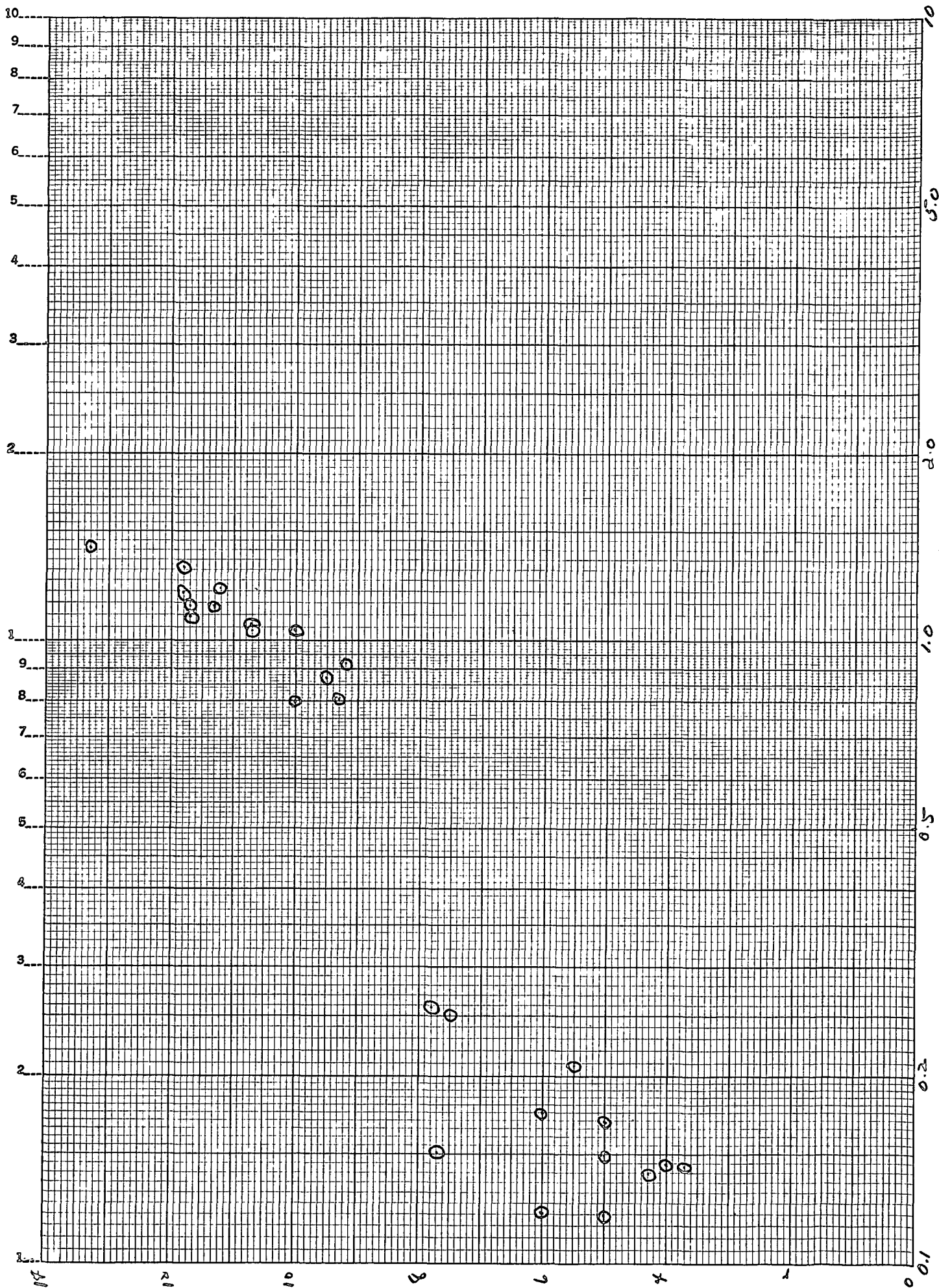
Life

Unpaired Fatigue

M.D

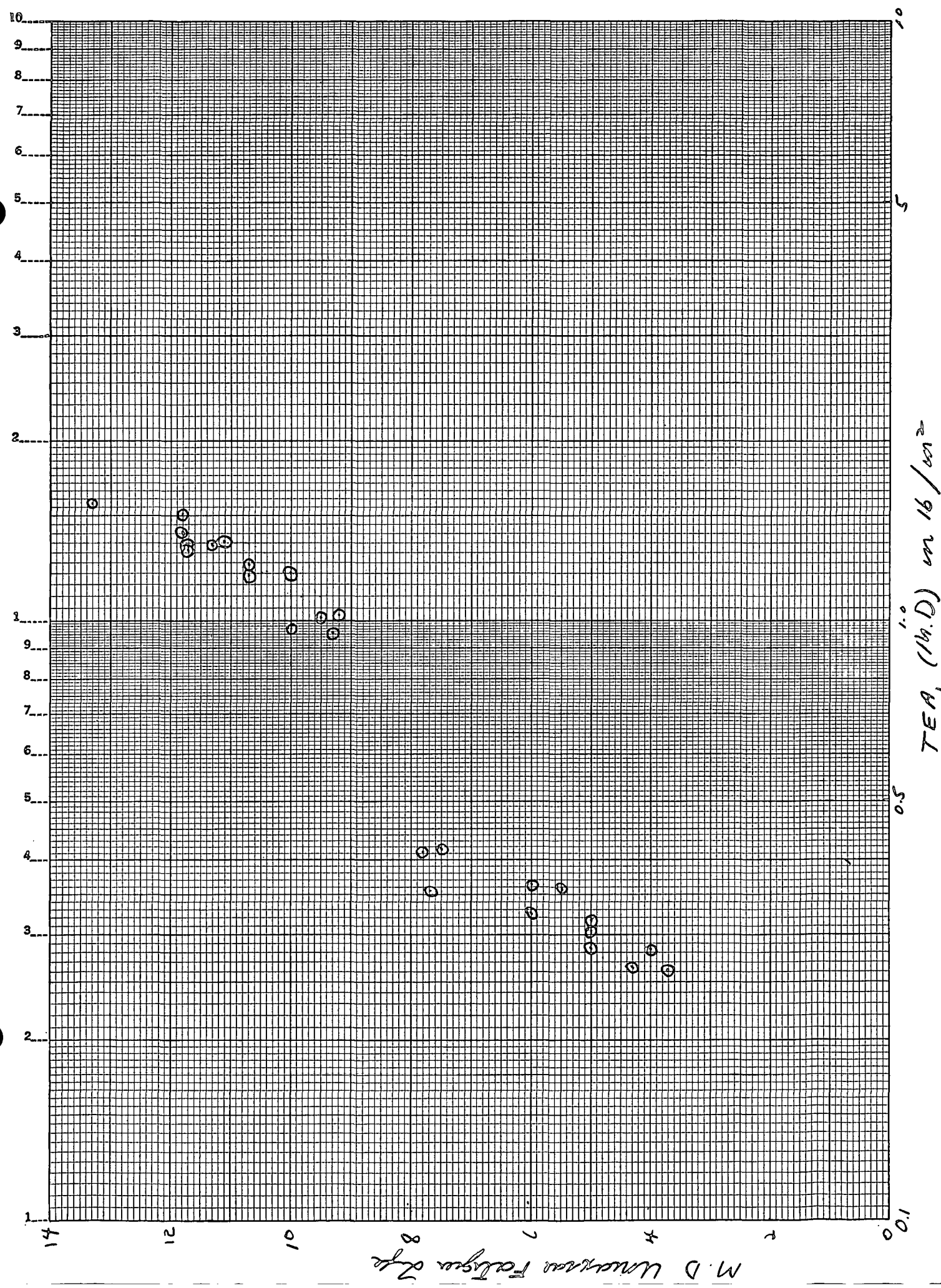


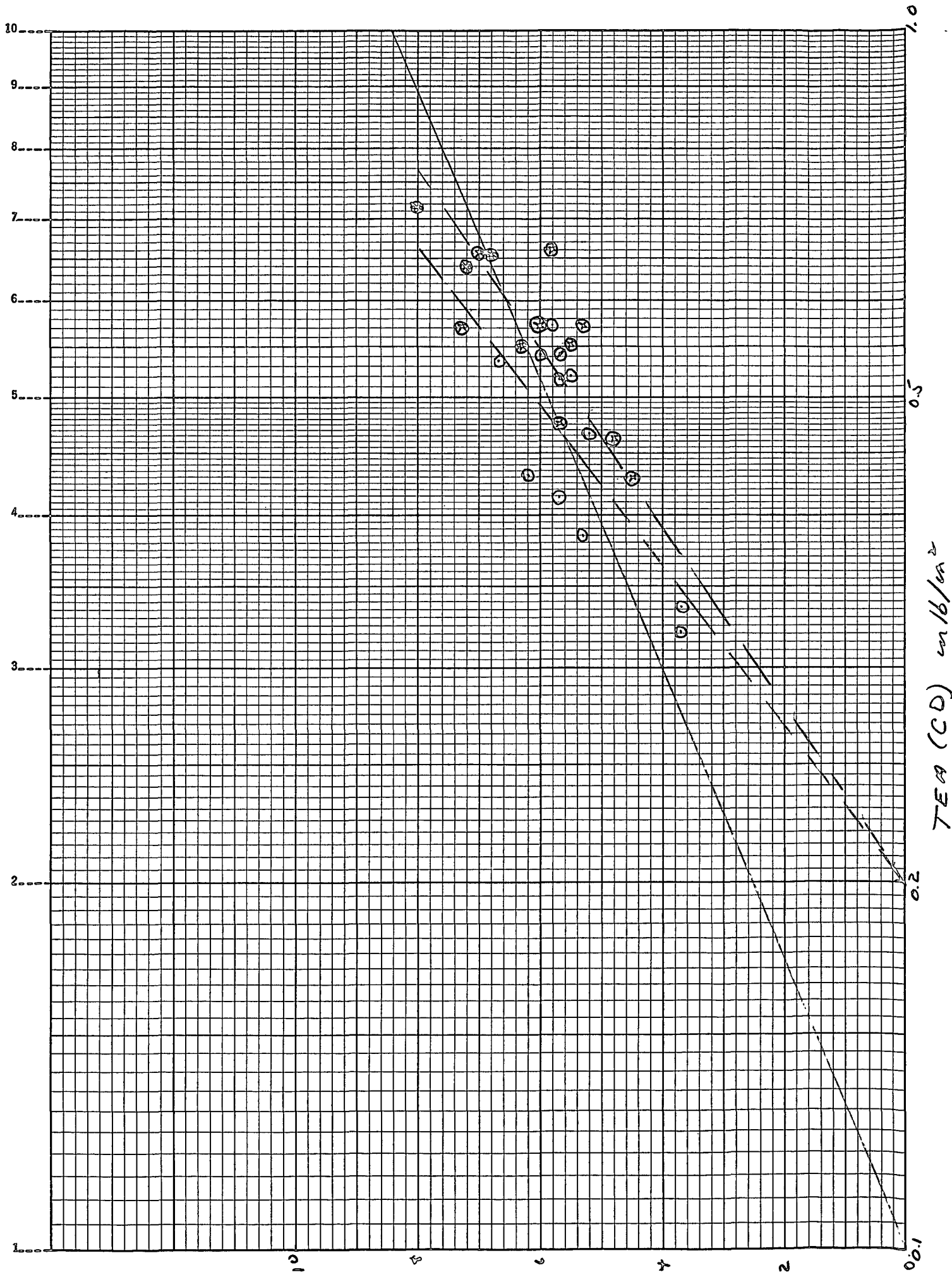
Stretch, % (M.D)



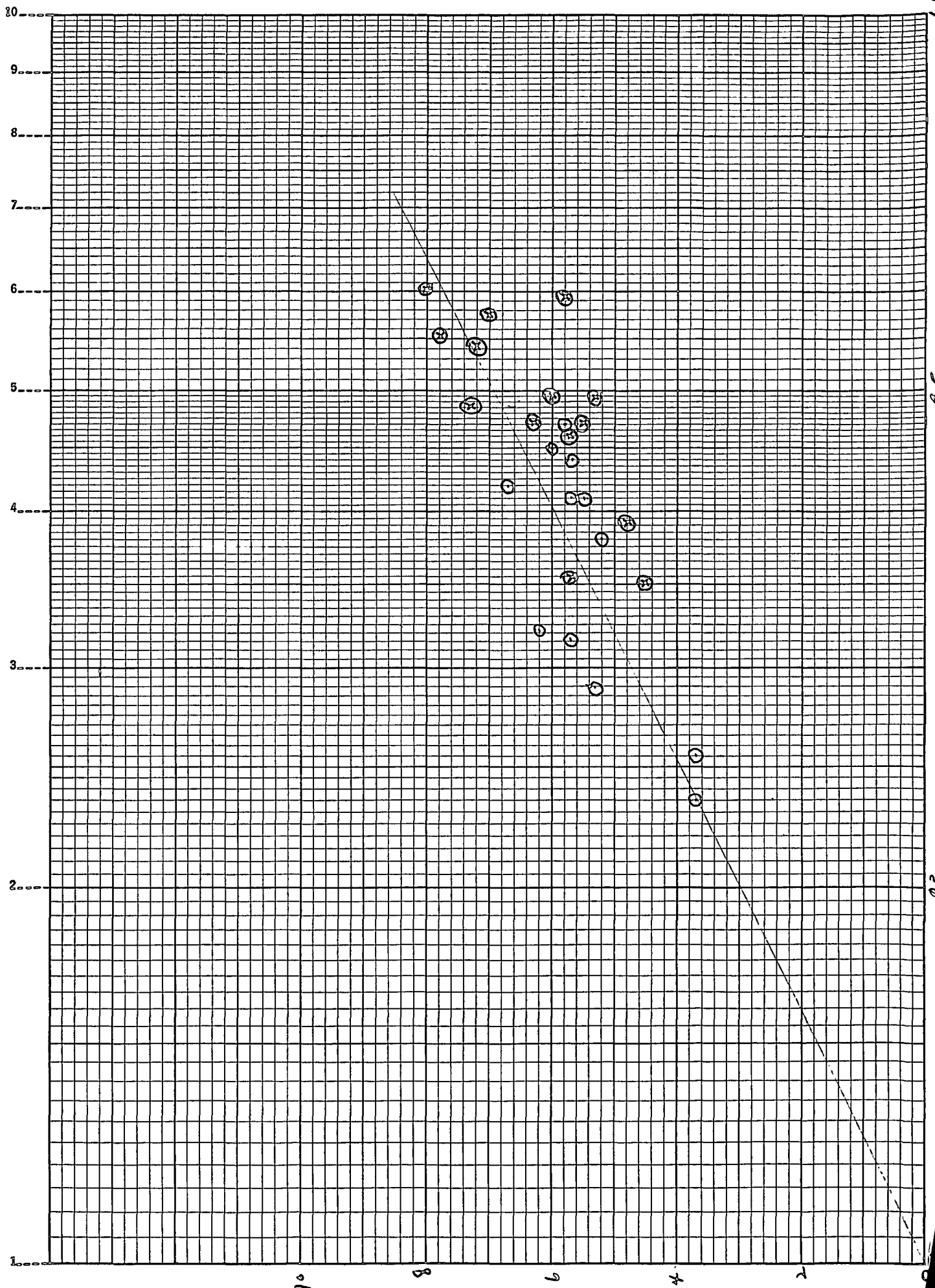
MD Plastic TERA, in 16/602

358-63 KEUFFEL & ESSER CO.  
Semi-Logarithmic, 2 Cycles X 10 to the 1/2 inch,  
5th lines accepted.  
MADE IN U.S.A.





TEA (CD)  $\text{cm}^2/\text{hr}^2$



0.2  
 0.5  
 1.0  
 PLASTIC TEA (CD) .  $m_{16}/100^2$



Relationships between sack freight life and failure points for regular sacks.

Average sack inches

Row	Grand Average	AB	c	D	E	FS	FX	G	H
AA	12	462	(10) 268	(15) 684	(17) 208	-- (6)	-- (6)	264	-- (6)
BB	10	<del>410</del> (9) 217	(8) 411	(12) 387	(18) 411	372 (4)	372 (1)	361 (5)	239 (1)
CC	11	649	(9) 224	(9) 284	(13) 440	370 (2)	321 (2)	363 (4)	-- (6)
DD	7	389	(11) 249	(12) -- (6)	333	-- (6)	132 (1)	164 (3)	95 (1)
EE	8	296	(12) 141	(10) 132	(14) 158	-- (6)	-- (6)	106 (3)	-- (6)
FF	9	314	(11) 137	(11) 183	(12) -- (6)	258 (3)	244 (1)	132 (2)	-- (6)
GG	11	459	(8) 283	(11) 40	(17) 348	-- (6)	185 (2)	317 (7)	-- (6)
HH	12	362	(8) 231	(12) -- (6)	(16) 456	156 (2)	180 (1)	199 (6)	182 (1)
II	3	406	(13) 231	(10) 334	(17) 432	-- (6)	234 (1)	411 (4)	-- (6)
JJ	1	643	(9) 311	(6) 234	(14) 586	-- (6)	-- (6)	466 (6)	54 (1)
KK	1	320	(11) 246	(8) 149	(14) 231	-- (6)	359 (5)	256 (4)	356 (1)
LL	2	433	(5) 230	(11) 135	(17) 330	-- (6)	90 (1)	264 (4)	-- (6)
Mean	8.2	0	12	8	2	1	6	8	4
Standard Deviation	12.18	0	2	8	5	3	3	4	1

Mean and Standard Deviation  
 8.2 12.18  
 2 8 5 3 4 1

Note: No in Parenthesis equal No of sacks exhibiting the particular type of failure

elationship between rank, height, dips and fitness of person for climbing rocks.

Progressive Weight for day, 1951  
Fardus Station as reported in Report 2

Run	Speed Average	A 1 + B	C	D	E	F	G	H
10	955	1124(18)	534	1010(10)	534	534	534	534
11	987	1041(15)	714(5)	534	534	534	534	534
12	1144	1022(14)	420(6)	1000(1)	534	534	534	534
13	781	894(21)	420(6)	1000(1)	534	534	534	534
14	1023	1018(20)	868(4)	404	534	534	534	534
15	1288	1044(19)	420(6)	204	534	534	534	534
16	438	501(16)	258(1)	434	534	534	534	534
17	565	610(22)	388(3)	384	534	534	534	534
18	585	623(21)	388(3)	384	534	534	534	534
19	1038	1189(27)	868(4)	567(1)	534	534	534	534
20	807	813(23)	520(1)	534	534	534	534	534
21	650	648(24)	520(1)	534	534	534	534	534
22	727	734(25)	620(2)	534	534	534	534	534
23	951	1053(27)	867(1)	534	534	534	534	534
24		3	13	6	2	1	1	1
25		11	0	9	4	1	1	1

was lower than average  
" " higher

Note: No parameters reported. No of males and the station the parameter type of weather.

TABLE VI

FRAG TEST RESULTS FOR REGULAR AND EXTENSIBLE PAPERS

Run	Frag, no. of safe drops	
	In	Cross
Regular		
AA	35	86
BB	14	32
CC	12	46
DD	46	28
EE	18	18
FF	8	13
GG	15	52
HH	10	31
II	24	41
JJ	81	106
KK	8	47
LL	14	46
Extensible		
MM	167	70
NN	187	32
OO	245	27
PP	112	59
QQ	139	53
RR	196	43
SS	53	26
TT	67	24
UU	82	21
VV	210	49
WW	51	12
XX	112	20
YY	113	37
ZZ	124	25

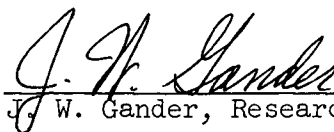
Note: Each entry is the average of 72 tests.

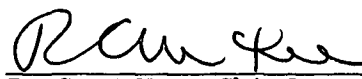


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THE INSTITUTE OF PAPER CHEMISTRY

  
J. W. Gander, Research Aide

  
R. C. McKee, Chief  
Container Section

APPENDIX

TABLE VII  
INSTON FATIGUE LIFE  
(50% R.H.)

		Fatigue Life, no. of energy applications															
		In	Cross	In	Cross	In	Cross	In	Cross	In	Cross	In	Cross	In	Cross	In	Cross
		Run AA		Run BB		Run CC		Run DD		Run EE		Run FF		Run GG		Run HH	
Start	8	2 (4) <sup>a</sup>	5	5	5	5	5	8	4	6	4	4	4	9	5	5	5
	8	7 (8)	6	6	6	6	6	8	5	3	4	4	4	4	4	4	4
	8	3 (6)	5	4	4	4	4	7	5	2	2	4	4	8	2	2	7
End	7	8	5	6	4	4	4	8	7	5	4	3	6	7	5	6	6
	7	6	6	5	4	7	6	6	6	2	4	3	4	6	4	5	5
	7	7	6	7	5	7	5	9	5	4	4	4	4	6	4	7	7
Av.	7.5	5.7	5.5	5.2	5.0	6.0	5.3	7.7	5.0	3.7	3.7	3.7	5.0	6.7	4.0	4.0	5.7
		Run II		Run JJ		Run KK		Run LL		Run MM		Run NN		Run OO		Run PP	
Start	6	7	9 (8) <sup>a</sup>	4	4	5	6	6	10	9	10	5	12	8	9	5	5
	5	7	7 (8)	4	4	5	6	6	9	7	9	6	12	7	10	7	7
	5	7	7 (8)	5	5	5	7	7	8	5	11	9	12	7	10	7	7
End	8	3	7 (8)	4	4	6	3	3	10	5	10	6	13	6	9	6	6
	5	7	8 (7)	5	5	6	6	6	9	9	11	9	11	7	9	6	6
	7	6	7 (5)	4	4	6	6	6	10	8	11	8	11	6	10	3	3
Av.	6.0	6.2	7.8	4.3	4.3	5.5	5.7	6.0	9.3	8.0	10.7	7.2	11.8	6.8	9.5	5.7	5.7
		Run QQ		Run RR		Run SS		Run TT		Run UU		Run VV		Run WW		Run XX	
Start	12	7	13	11	11	3	5	11	11	7	11	6	10	6	13	4	4
	12	4	14	9	11	4	5	11	10	7	12	8	9	6	12	3	3
	12	6	13	10	10	6	5	10	12	7	12	7	9	6	11	6	6
End	10	6	13	11	11	5	7	11	11	7	11	5	8	5	12	3	3
	12	6	13	8	10	6	6	10	13	8	11	8	9	5	11	6	6
	12	7	14	11	11	5	4	11	14	8	11	8	10	6	11	5	5
Av.	11.7	6.0	13.3	10.0	10.7	4.8	5.3	10.7	11.8	7.3	11.3	7.0	9.2	5.7	11.7	4.5	4.5
		Run YY		Run ZZ													
Start	10	5	11	11	5												
	11	6	11	11	5												
	9	7	13	11	8												
End	10	3	11	11	6												
	10	6	10	10	8												
	10	6	11	11	6												
Av.	10.0	5.5	11.2	11.2	6.3												

<sup>a</sup>Values in parentheses are retests; included in average.

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